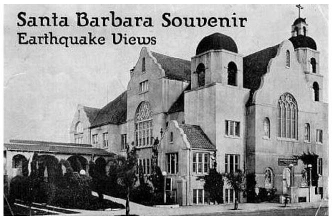

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MICROTREMOR BASED JOINT INVERSION FOR ESTIMATING S-WAVE VELOCITY PROFILE OF TWO-DIMENSIONAL IRREGULAR GROUND

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ABSTRACT

In order to evaluate the site effect, one of the key parameters is to obtain the S-wave velocity profile. Microtremor measurement is one of the most popular ways to investigate the site effect and has been conducted extensively because of its cost effectiveness. The joint inversion of microtremor array data, i.e. dispersion curve, and horizontal to vertical ratio (H/V spectrum) to obtain the S-wave velocity profile has been successfully used by several researchers. However, a difficulty arises in the case of an irregular ground where the parallel layer assumption is not established. In this paper, a two-dimensional S-wave velocity profile inversion method is introduced in which H/V spectra and dispersion curves are used. In the inversion process, a 2.5-dimensional analysis of the wave-field due to incident surface waves was conducted as a forward analysis assuming that microtremor is composed of waves coming from a various directions. It was found from the study that the proposed method shows a possibility to get a very positive result for the irregular ground. It is also found, however, that enough information about the source such as incoming direction and its intensity is necessary for the proposed method.

INTRODUCTION

It is well known that the evaluation of site effect is one of the key components when considering earthquake disaster prevention. In order for the evaluation of the site effect, it is important to obtain the S-wave velocity profile. There are a number of methods available that are used for this purpose. Among others, microtremor measurement is one of the most popular ways and has been conducted extensively because of its cost effectiveness. The joint inversion of seismic array data (i.e. dispersion curve (e.g., Aki [1957]; Capon [1969]; Cho *et al.* [2006])) and horizontal to vertical amplitude ratio (H/V spectrum (Nakamura [1989]; Tokimatsu *et al*. [1998]; Arai *et al*. [2004])) in order to obtain the S-wave velocity profile has been successfully used by several authors (e.g., Arai *et al*. [2005]). However, a difficulty arises in the case of an irregular ground where the parallel layer assumption is not established. Uebayashi [2003] has pointed out that H/V spectrum is influenced by the irregularity of a ground in the low frequency range. Nakagawa and Nakai [2010b] also revealed that the irregularity affects both H/V spectrum and dispersion curve in the high frequency range. These studies imply that multi-dimensional analysis is necessary for the inversion analysis based on microtremor measurements such as H/V spectra and dispersion curves when conducted at the locations near the irregularity of a ground. Uebayashi [2006] has conducted the inversion of microtremor H/V spectra for determining the boundary shape of a basin structure by numerical experiments using two-dimensional finite element analysis of a wave field due to incident fundamental Rayleigh waves from both left and right sides. However, he does not consider the Love wave components (out-of-plane wave motions) that may be involved in microtremor H/V spectral ratio. In addition, the dispersion curve is not considered in his inversion process. In order to apply the microtremor-based inversion to a practical problem, it is important to take into account other factors such as Love waves, higher mode incidence and various incoming directions. In this article, the authors propose a joint inversion scheme that can be used for estimating two-dimensional S-wave velocity structure based on microtremor H/V spectra and dispersion curves calculated by 2.5-dimensional analysis (Nakagawa *et al.* [2010a, b]). The validity of the present method is examined by a numerical experiment.

JOINT INVERSION METHODOLOGY

2.5-dimensional analysis

In the two-dimensional problem, a plane strain condition is assumed for both the ground and wave propagation. Since the SH wave field and the P-SV wave field are independent in this problem, the coupling of the Love wave and the Rayleigh wave cannot be considered. It is well accepted, however, that microtremor is coming from a variety of directions. In order to consider the variation of the direction of incoming surface waves, the method can be extended in such a way that, while the ground is in the plane strain condition, the incoming wave travels in a direction that is not perpendicular to the out-of-plane direction of the ground. This problem is often called a 2.5-dimensional problem. In this study, the forward analysis was done by the hybrid method in which the thin layered element method (TLM) and the finite element method (FEM) are combined (e.g., Nakagawa *et al.* [2010a, b]). The amplitude of incident Rayleigh wave is determined by considering medium response (Harkrider [1964]) obtained from the eigenvalue analysis of two-dimensional thin layered element method (Nakagawa *et al.* [2010a, b]). In the case of Love wave incidence, the amplitude was adjusted in such a way that the ratio between the Rayleigh and the Love wave components in the horizontal direction is set to 0.7 regardless of the frequency (Tokimatsu *et al.* [1998]; Arai *et al.* [2004]).

Joint Inversion Analysis

The joint inversion analysis is based on the nonlinear least square method. If the number of observed H/V spectral ratios, $H/V_i^{(obs)}$, and the phase velocities of vertical motion, $c_i^{(obs)}$, are given as n_{HV} and n_R, respectively, the goal of the inversion process is to find a S-wave velocity structure that satisfies the following least-square equation (Arai *et al.* [2005]):

$$F = \frac{w_{HV}}{n_{HV}} \sum_{i=1}^{n_{HV}} \left(\frac{(H/V)_i^{(obs)} - (H/V)_i^{(cal)}}{(H/V)_i^{(obs)}} \right)^2 + \frac{w_R}{n_R} \sum_{i=1}^{n_R} \left(\frac{c_i^{(obs)} - c_i^{(cal)}}{c_i^{(obs)}} \right)^2 \rightarrow \min . \quad (1)$$

where $H/V_i^{(cal)}$ and $c_i^{(cal)}$ are the H/V spectral ratio and the phase velocity computed from the one-dimensional or 2.5-dimensional analysis result. w_{HV} and w_R are the weighting factors for the H/V spectral ratio and the dispersion curve. (In this study, $w_{HV} = w_R = 1$) The Jacobians of the linearized least-square scheme were obtained numerically.

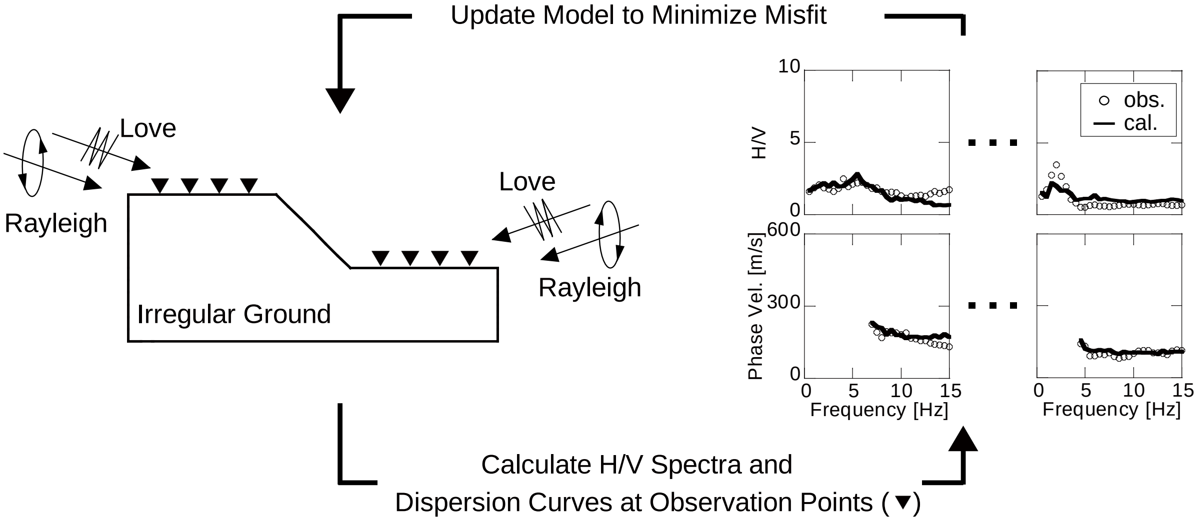


Fig. 1. Schematic diagram showing nonlinear joint inversion in an irregular ground by the use of H/V spectra and dispersion curves at observation points (▼). The iteration is repeated until the root mean of the sum of squares of the normalized misfit, F in Eq. (1), is converged to an acceptable small value, and the S-wave structure is then determined.

Calculation of H/V Spectra and phase velocities from the 2.5-dimensional analysis

H/V Spectrum. In the computation of H/V spectral ratio, summation was done in terms of the power of each displacement amplitude for all cases of incident surface (Rayleigh and Love) waves.

$$H/V = \sqrt{\sum_{l=1}^{W} \left|u_x\right|_l^2 + \sum_{l=1}^{W} \left|u_y\right|_l^2} \Bigg/ \sqrt{\sum_{l=1}^{W} \left|u_z\right|_l^2} \tag{2}$$

where W is the number of incident surface (Rayleigh and Love) waves. u represents the displacement at an observation point. The suffix indicates each coordinate. In the above computation, each component has the same angle of incidence.

Phase Velocity. In the analysis, the phase velocity was also obtained from vertical components by referring to the centerless circular array (CCA) method (Cho *et al.* [2006]). Calculation of phase velocities from the 2.5-dimennsional analysis is briefly explained below. Suppose we deploy a circular array of radius r as shown in Fig. 2, and let the vertical component of microtremor be denoted by D_j, where suffix j indicates the sensor number located on the array circle. For the frequency, f, the average value along the circumference is given by:

$$D_{ave}(f) = \frac{1}{M} \sum_{j=1}^{M} D_j(f) \tag{3}$$

and, a weighted average of D_j over the azimuth angle θ is computed as:

$$D_{w-ave}(f) = \frac{1}{M} \sum_{j=1}^{M} D_j(f) \exp(2\pi ji/M) \tag{4}$$

By computing the power spectral ratio, we can obtain the CCA coefficient, ρ_{cca}, using the following equation:

$$\rho_{cca}(f) = \sum_{l=1}^{W} \left|D_{ave}(f)\right|_l^2 \Bigg/ \sum_{l=1}^{W} \left|D_{w-ave}(f)\right|_l^2 \tag{5}$$

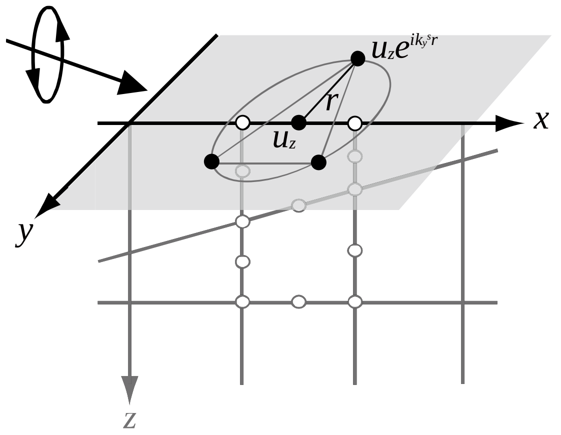


Fig. 2. Estimation of phase velocities from the 2.5-dimensional analysis. Open circles (○) indicate the nodal points of finite elements. Closed circles (●) are observation points for calculating phase velocity, which can be obtained from nodal displacements as a function of the wave-number in the y-direction, k_y .

If the radius r is very small compared to a target wave-length, ρ_{cca} is related to the phase velocity, c, by the following expression (Tada *et al.* [2007]):

$$c = \pi f r \sqrt{\frac{2+\rho_{cca}}{1+\varepsilon/M - \varepsilon\rho_{cca}/M}} \tag{6}$$

where ε is the noise-to-signal (NS) ratio (reciprocal of signal-to-noise ratio). Basically, the center of the array circle is not needed in the CCA method for determining the phase velocity, hence its naming is "centerless". But in order to obtain the NS ratio, ε, it is necessary to use the center component. ε is defined by the following equation:

$$\varepsilon = \frac{\left(-B - \sqrt{B^2 - 4AC}\right)}{2A} \tag{7}$$

where

$$\left.\begin{aligned} A &= -\rho_{spac}^2 \\ B &= \frac{\rho_{spac}^2}{coh^2} - 2\rho_{spac}^2 - \frac{1}{M} \\ C &= \rho_{spac}^2\left(\frac{1}{coh^2} - 1\right) \end{aligned}\right\} \tag{8}$$

where, ρ_{spac} is the SPAC (SPatial AutoCorrelation) coefficient derived by Aki [1957], and is calculated from the cross spectrum between averaged value along the circle, D_{ave}, and its center divided by the power spectrum of a center record. This method of computing SPAC coefficients has been utilized, although alternative approaches are proposed in which geometrical average of power spectra at all the points including the center in addition to the circumferential points as the denominator in the calculation of the coefficient (Okada *et al*. [1987]).

NUMERICAL EXPERIMENT

In order to show the validity of the proposed method, a numerical experiment was carried out where H/V spectra and phase velocities computed from a hypothetical target model (Fig. 3) were used as observed data. In this study, S-wave velocities of the finite elements are the variable parameters in the joint inversion process. P-wave velocities, densities, and damping factors were fixed as constant values.

Analysis Model and Target Data for Joint Inversion

In this article, an irregular ground with a slope is considered, as shown in Fig. 3. The soil consists of two layers and the boundary between the layers is parallel to the ground surface. The underlying layer is assumed as a half-space. The angle of inclination of the slope is 45° and the height is 12m, same as the thickness of the surface layer. As shown in Fig. 3, the surface layer is comprised of 6 layers of quadratic elements, while 19 layers of elements are allotted to the underlying layer. The fixed condition is assumed at the bottom boundary but its depth varies according to the frequency in order for the half-space approximation to be made, following the relationship: $H = 4V_s/f$, where V_s is the shear wave velocity of the underlying layer. In order to suppress body wave reflection from the bottom boundary, relatively large damping of 30% to 50 % is assumed in the lower part of the model. Incident angles, ϕ, considered are 0°, 30°, 45°, 60°, 120°, 135°, 150° and 180°. Angles greater than 90° represent incidence from the right side of the finite element region. It is noted that the results due to incidence from back side and from front side are the same (e.g., –30° and 30°) in the 2.5-dimensional analysis. According to the previous study (Nakagawa *et al.* [2010b]), phase velocities from observation data show a tendency of accuracy degradation in the vicinity of the H/V spectral peak. Thus, the target phase velocity for the inversion is limited to the frequency range of 6 to 15 Hz.

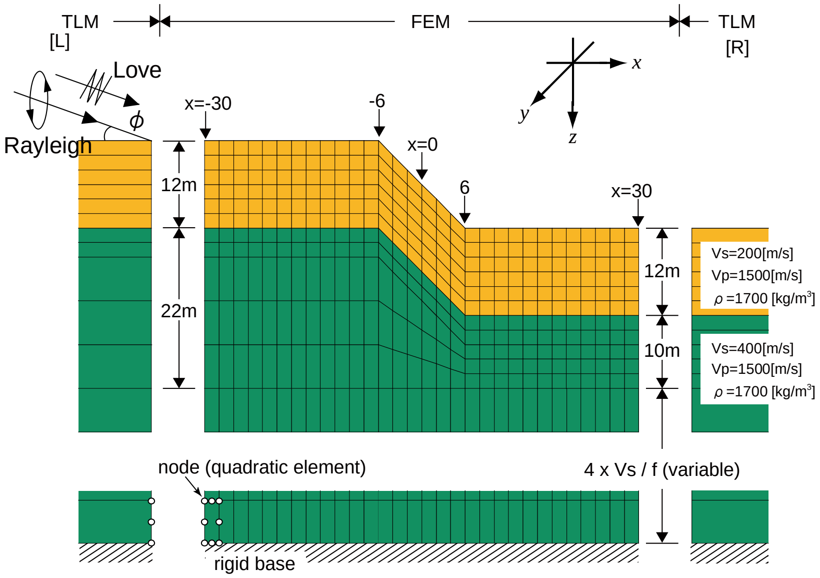
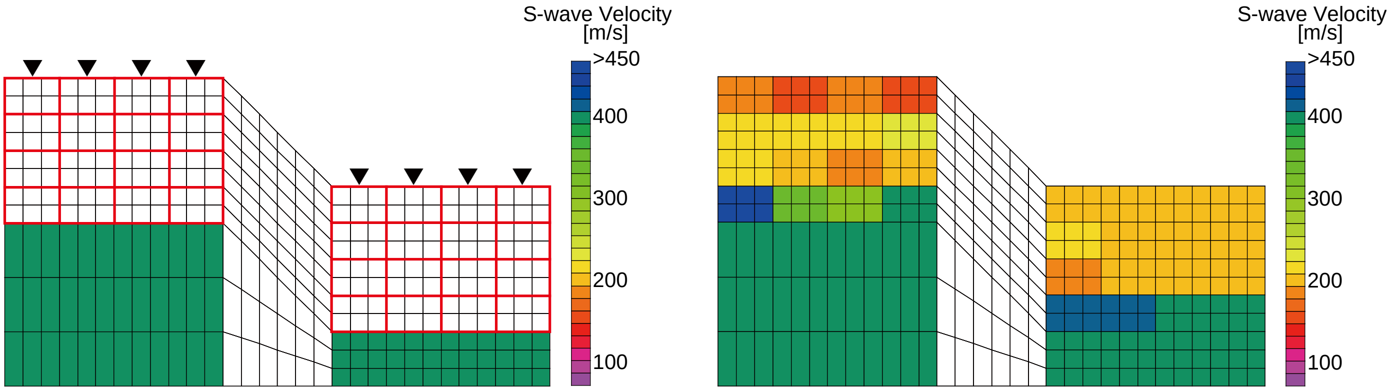


Fig. 3. Analysis model (i.e. true assumed model) for the joint inversion. Number of elements is 752. Number of degree of freedoms is 7083.

One-dimensional Joint Inversion

First, an initial model for two-dimensional joint inversion was made by the use of a one-dimensional joint inversion scheme (Arai *et al*. [2005]). Since the soil model consists of 5 layers (i.e., 4 surface layers and a half-space), variable parameters are S-wave velocities of the surface layers. The thickness of each surface layer is 4m, and the S-wave velocity of a half-space was assumed to be known. Initial model for one-dimensional inversion is set to the true model. Fig. 4 and 5 show the results of one-dimensional joint inversion. From these figures, it is seen that the one-dimensional parallel layer assumption gives a fairly good estimation. However, its prediction may involve errors in the location near the slope, especially, in the upper part of the slope. It appears that an irregular ground (slope in this study) affects both H/V spectra and dispersion curves. In short, this is the effect of interference between incident waves and generated (reflection, refraction, diffraction or scattering) waves from the slope. At some locations, i.e. $x = -15, -21$, at the upper part of the slope ground, estimation gives a softer S-wave velocity profile for the underlying layer. This may result from lower value of phase velocity of the target data in these locations for the frequency range of 6 to 9 Hz. It is noted that one-dimensional inversion in the lower part of the slope ground gives fairly good estimation to the true model in this study.



(a) Observation points (▼) and variable parameters (□) for one-dimensional joint inversion

(b) One-dimensional joint inversion result

Fig. 4. (a) Observation points in this study and (b) estimated model using 1D inversion. The upside-down triangles (▼) indicate observation points. Colors represent S-wave velocity. White color means undecided value.

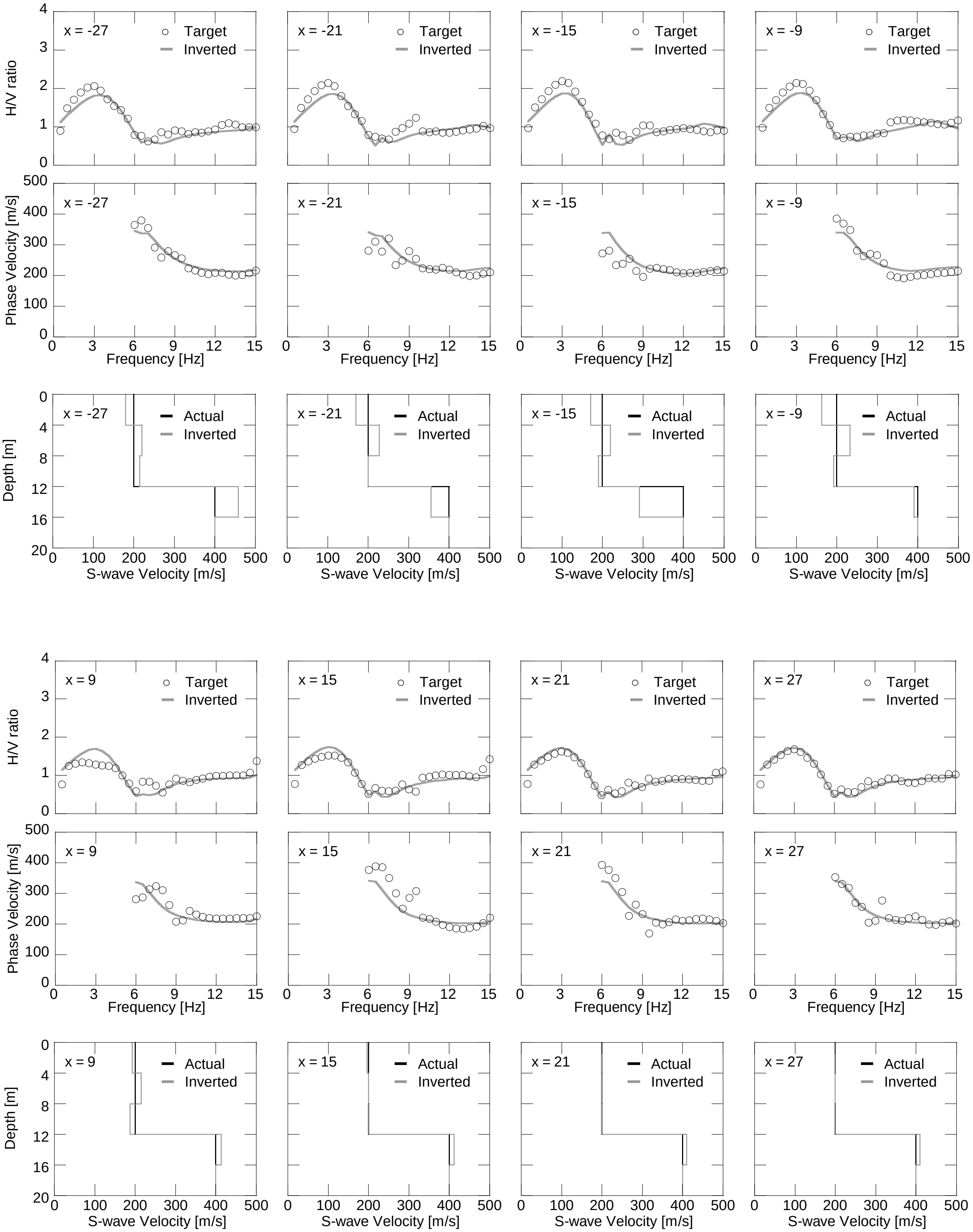


Fig. 5. One-dimensional joint inversion analysis results (H/V spectrum, dispersion curve and S-wave velocity profile). Open circles denotes the target. Gray solid lines are inverted values. Black lines indicate the actual S-wave velocity profile.

2.5-dimensional Joint Inversion

In this study, an initial model for two-dimensional joint inversion was made by the use of one-dimensional joint inversion scheme (Arai *et al*. [2005]). In the slope part, the initial values were computed as the average of top and bottom part of the slope. Number of parameters is 36. Fig. 6 shows the inversion process of the 2.5-dimensional analysis. It is found that the proposed inversion method gives quite a promising result. This result indicates that the proposed method has the possibility of determining the S-wave velocity structure in an irregular ground with a relatively high accuracy. It also indicates the possibility to find the S-wave velocity without observation points in the slope. Next, we conduct the inversion in order to consider errors in observation data. The inversion result for the case of target data which involve 10% errors is shown in Fig.7. Since the final estimation is close to the true model, this result implies the robustness of the proposed method.

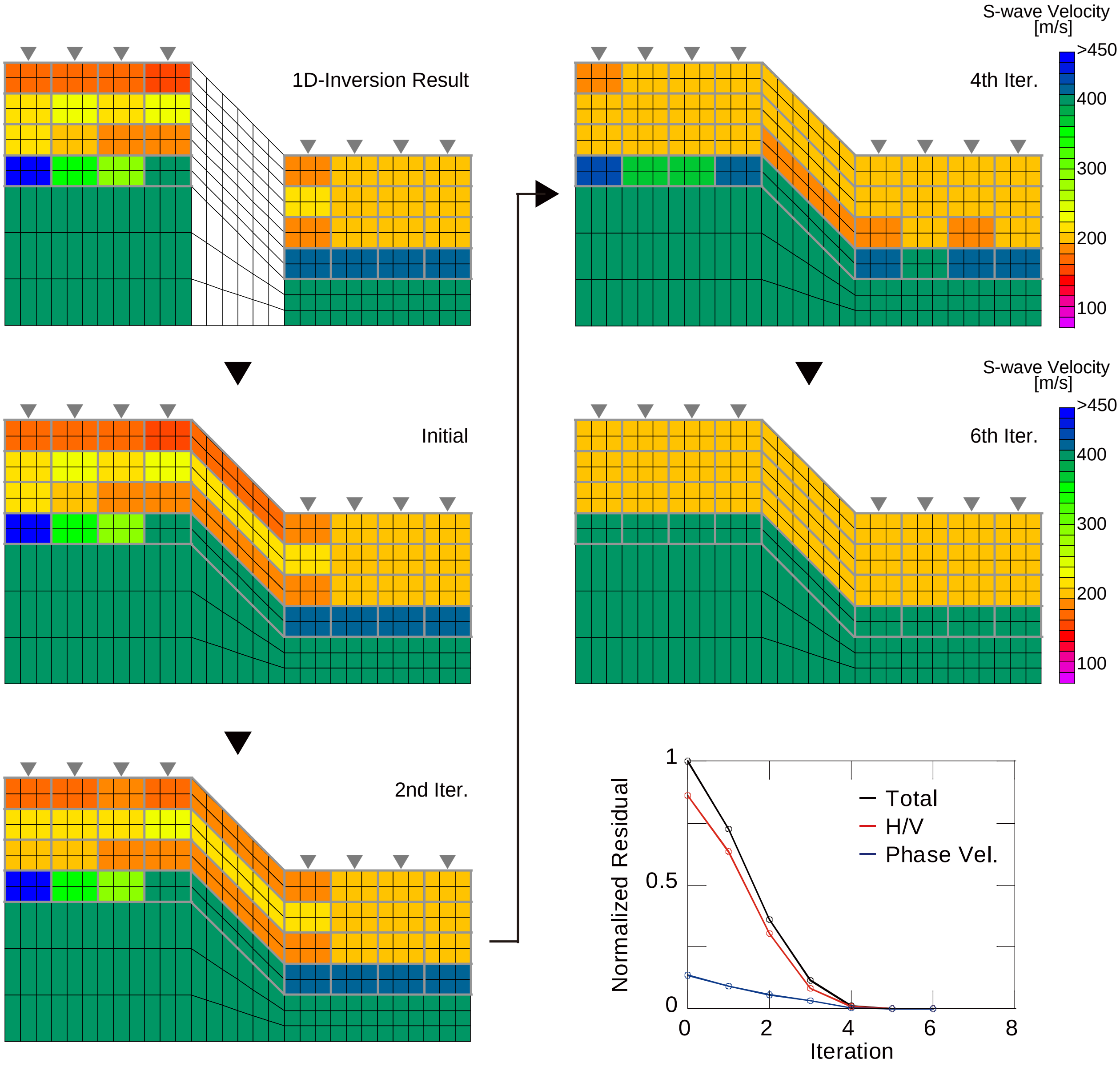


Fig. 6. Iteration process of 2.5-dimensional inversion result. Colors represent the S-wave velocity. The gray rectangles (□) denote variable parameters. The gray upside-down triangles (▼) indicate observation points. The graph on the bottom right shows the change of residuals with iterations. Black, red and blue lines denote the total, H/V spectral ratio and dispersion curve errors, respectively.

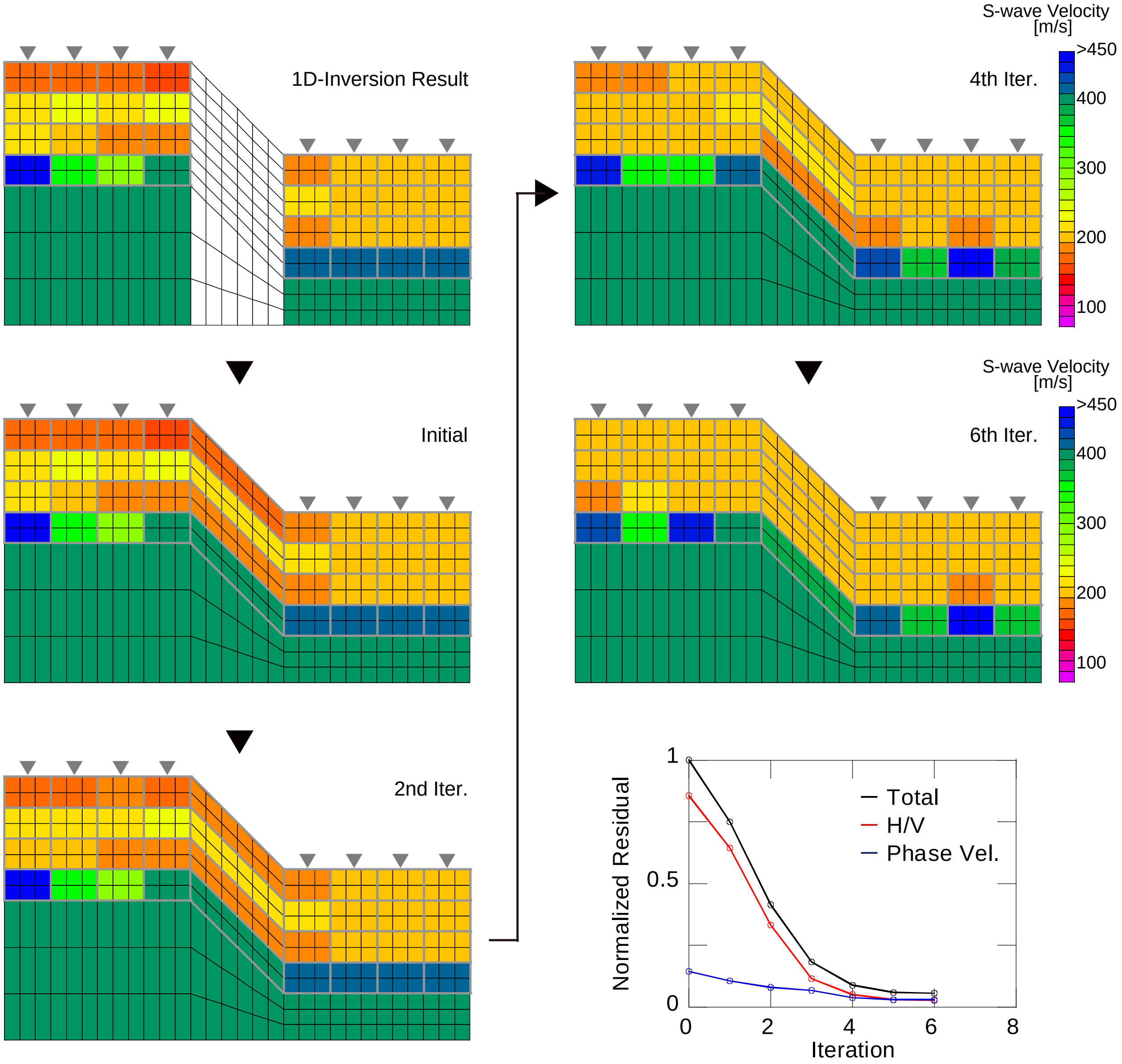


Fig. 7. Iteration process of 2.5-dimensional inversion in the case of the target (observation) data with errors of 10%. Colors represent the S-wave velocity. The gray rectangles (□) denote variable parameters. The gray upside-down triangles (▼) indicate observation points. The graph on the bottom right shows the change of residuals with iterations. Black, red and blue lines denote the total, H/V spectral ratio and dispersion curve errors, respectively.

However, inversion results shown in Figs. 6 and 7 indicate some limitations in that there is a room for modification in the two-dimensional inversion. One of the problems is the source of microtremors. If the azimuth angles of incident surface waves differ from the true input, it is just obvious that we cannot obtain a good result. Fig. 8 shows the inversion result for different types of incident angles. The azimuth angles of incident surface waves are 0° and 180° (i.e. two-dimensional problem). The initial model for inversion is the same as previous inversion (see Fig. 6). Estimated S-wave velocity profiles are shown in Fig. 8 which shows the fact that two-dimensional joint inversion gives a poor result compared with one-dimensional inversion. This result indicates that it is important to take into account source information for determining an S-wave velocity structure in the multi-dimensional inversion. In practice, microtremors are coming from a variety of directions. From this viewpoint of the two-dimensional analysis cannot be good at everything for the multi-dimensional inversion using microtremor H/V spectra and dispersion curves. In order to obtain a good result for an irregular ground, it is necessary to take into account at least three-dimensional wave-field. In order to do that, we should use the 2.5-dimensional analysis or three-dimensional analysis (e.g. Nakai *et al.* [2011a, b]) for forward analysis of the inversion process.

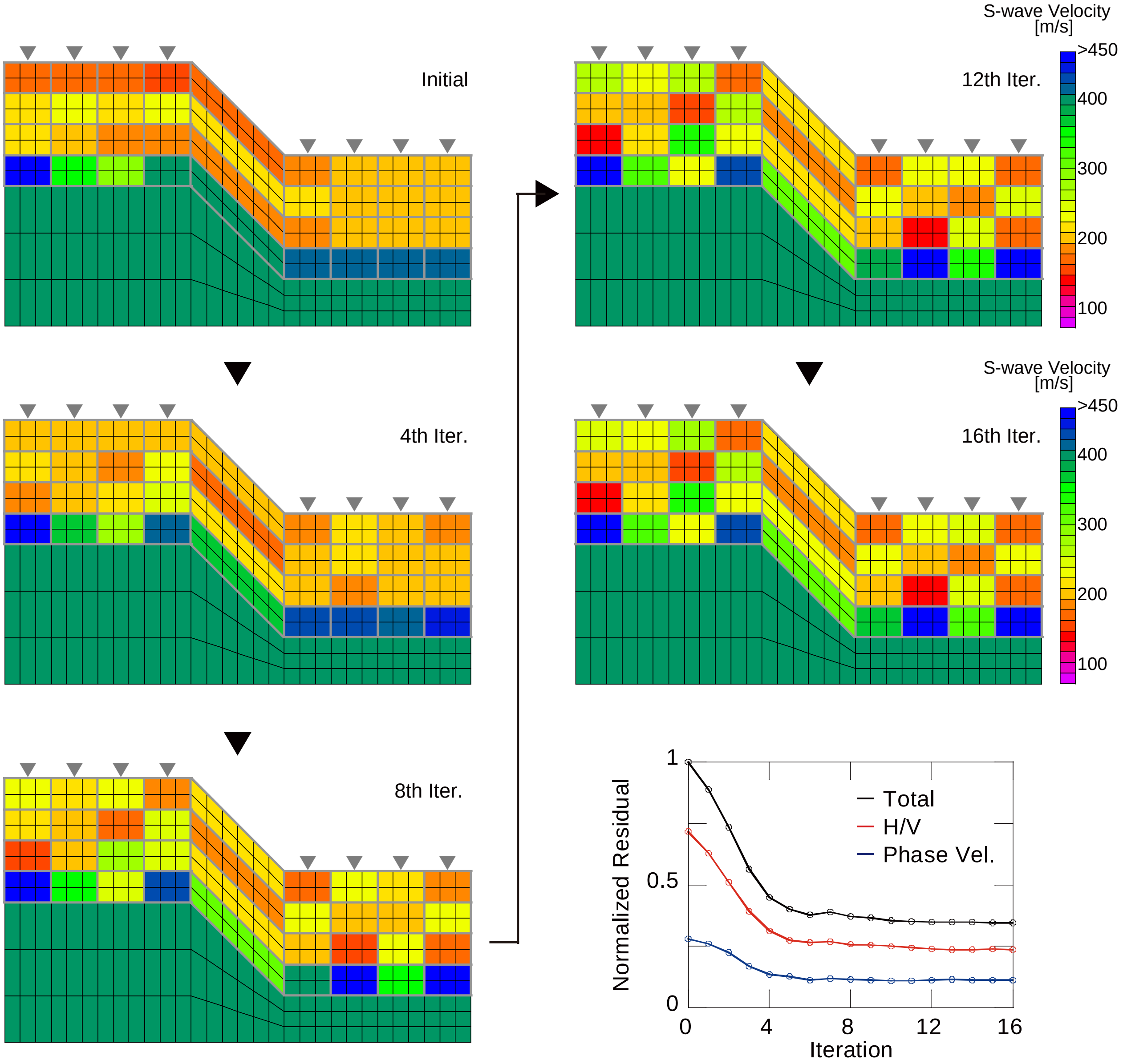


Fig. 8. Iteration process of two-dimensional inversion result. Colors represent the S-wave velocity. The gray rectangles (□) denote variable parameters. The gray upside-down triangles (▼) indicate observation points. The graph on the bottom right shows the change of residuals with iterations. Black, red and blue lines denote the total, H/V spectral ratio and dispersion curve errors, respectively.

CONCLUSIONS

In this paper, a two-dimensional S-wave velocity inversion method was introduced by the use of H/V spectra and dispersion curves. Based on the numerical analysis, the proposed method shows the possibility to get a positive result in an irregular ground. However, in order to apply this method to a practical problem, there are still some problems to be addressed. One of the problems is a lack of information about the input source (such as incident angles and incident modes). It is also difficult how to set the parameters for inversion because most of the calculation time is spent for obtaining sensitivity functions. In order to solve these problems, it will need further study in an actual site where enough information is available.

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