SCALING OF SHORT-PERIOD SPECTRAL LEVEL OF ACCELERATION SOURCE SPECTRA FOR AFTERSHOCKS AND FORESHOCKS OF THE 2011 OFF THE PACIFIC COAST OF TOHOKU EARTHQUAKE

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ABSTRACT

We estimate short-period spectral levels $A$ of acceleration source spectra for aftershocks and foreshocks of the 2011 off the pacific coast of Tohoku earthquake ($M_w 9.0$) and previous earthquakes in and around this region, because $A$ is one of important parameters to predict strong motions using fault models by theoretical and semi-empirical methods. The $A$ of the interplate earthquakes estimated in this study are consistent with the Satoh's (2010) empirical relation for interplate earthquakes occurring on the boundary between the Pacific plate and the continental plate in Japan. It is also found that the $A$ of big earthquakes in Miyagi-ken oki tend to be large. The $A$ of the intraslab earthquake ($M_w 7.1$) in Miyagi-ken oki on 11 April, 2011 is consistent with the Satoh's (2004) empirical relation for intraslab earthquakes in Miyagi-ken oki and Fukushima-ken oki. The $A$ of normal earthquakes around the Fukushima coast which types of big earthquakes had rarely occurred in Japan are slightly smaller than or equal to strike slip earthquakes and significantly smaller than reverse earthquakes occurring in Japan. This tendency qualitatively agrees with ground motion attenuation relations derived from crustal earthquakes in the world by the NGA project.

INTRODUCTION

After the 2011 off the pacific coast of Tohoku earthquake, big aftershocks occurred in this region in Japan. The aftershocks are not only interplate earthquakes but also intraslab earthquakes and crustal earthquakes. In this study we estimate short-period spectral levels of acceleration source spectra for the aftershocks and foreshocks of the 2011 off the pacific coast of Tohoku earthquake and previous earthquakes in this region by a spectral inversion method. The short-period spectral levels of acceleration source spectra $A$ are the flat levels of acceleration source spectra in frequency range shorter than corner frequencies. Therefore $A$ is one of important parameters to predict strong motions using fault models by theoretical and semi-empirical methods.

Dan et al. (2001) proposed a empirical relation between $A$ and seismic moment $M_0$ from previous variable-slip rupture models for crustal earthquakes in the world. This relation is widely used to set fault parameters with asperities to predict strong motions of crustal earthquakes in Japan. Satoh (2010) proposed empirical relations between $A$ and $M_0$ for strike-slip and reverse earthquakes, respectively, by estimating $A$ from strong motion records in Japan by a spectral inversion method. In the Satoh's relations, the $A$ for strike-slip and reverse earthquakes are 0.64 times and 1.45 times of $A$ by Dan et al. (2001), respectively. Although big normal-faulting crustal earthquakes had not been rarely occurred in Japan, a normal earthquake with $M_w 6.6$ ($M_J 7.0$) and middle-sized several normal earthquakes occurred around the Fukushima coast after the 2011 off the pacific coast of Tohoku earthquake. Therefore we estimate $A$ of the normal earthquakes by a spectral inversion method, and compare the $M_0$-$A$ relations for strike-slip and reverse earthquakes by Satoh (2010).

Satoh (2010) proposed empirical relations between $A$ and $M_0$ for interplate earthquakes with $6.6 \leq M_w \leq 8.2$ occurring on the boundary between the Pacific plate and the continental plate in Japan. In this study, we estimate the $A$ of the interplate earthquakes with foreshocks, aftershocks and the other earthquakes not included in Satoh (2010) and clarify the differences of $M_0$-$A$ relations in source regions, such as Iwate-ken oki, Miyagi-ken oki, Fukushima-ken oki, and Ibaragi-ken oki.

It has been pointed out that $A$ of intraslab earthquakes are larger than those of interplate and crustal earthquakes (e.g., Satoh, 2004; Sasatani et al., 2006). We also estimate the $A$ of an intraslab earthquake with $M_w 7.1$ occurred in Miyagi-ken oki and compare the $A$ of previous intraslab earthquakes in Japan.
We use two sets of data, that is, subduction-zone earthquakes off the Pacific coast and crustal earthquakes around the Fukushima coast. The $A$ of subduction-zone earthquakes are estimated using the $Q$ value for path and empirical amplification factors estimated by a spectral inversion method (Satoh and Tatsumi, 2002) and the strong motion records at K-NET stations. On the other hand, $A$ of crustal earthquakes around the Fukushima coast are estimated by a spectral inversion method in this study because no $Q$ values had been estimated in this region.

Figure 1 shows epicenters by JMA and F-net CMT solutions by NIED of the subduction-zone earthquakes together with location of K-NET strong motion stations by NIED used in this study. The 2011 off the pacific coast of Tohoku earthquake S0 is not used in this study.

Table 1. List of 12 interplate plate earthquakes (S1 to S12) and an intraslab earthquake (A0)

<table>
<thead>
<tr>
<th>Name</th>
<th>Origin Time</th>
<th>Focal Depth</th>
<th>$M_1$</th>
<th>$M_0$</th>
<th>$M_w$</th>
<th>$A$</th>
<th>Source Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>10:06 JST 11 Oct 2003</td>
<td>26</td>
<td>6.8</td>
<td>1.42E+26</td>
<td>6.7</td>
<td>9.88E+25</td>
<td>Miyagi-ken oki</td>
</tr>
<tr>
<td>S3</td>
<td>01:45 JST 08 May 2008</td>
<td>35</td>
<td>7.0</td>
<td>1.97E+26</td>
<td>6.8</td>
<td>2.16E+26</td>
<td>Ibaragi-ken oki</td>
</tr>
<tr>
<td>S4</td>
<td>11:39 JST 19 Jul 2008</td>
<td>35</td>
<td>6.9</td>
<td>2.39E+26</td>
<td>6.9</td>
<td>1.31E+26</td>
<td>Fukushima-ken oki</td>
</tr>
<tr>
<td>S5</td>
<td>17:08 JST 14 Mar 2010</td>
<td>44</td>
<td>6.7</td>
<td>6.83E+25</td>
<td>6.5</td>
<td>1.50E+26</td>
<td>Fukushima-ken oki</td>
</tr>
<tr>
<td>S6</td>
<td>06:55 JST 05 Jul 2010</td>
<td>38</td>
<td>6.4</td>
<td>2.79E+25</td>
<td>6.2</td>
<td>1.24E+26</td>
<td>Iwate-ken oki</td>
</tr>
<tr>
<td>S7</td>
<td>11:45 JST 09 Mar 2011</td>
<td>23</td>
<td>7.3</td>
<td>7.97E+26</td>
<td>7.2</td>
<td>5.97E+26</td>
<td>Miyagi-ken oki</td>
</tr>
<tr>
<td>S8</td>
<td>06:23 JST 10 Mar 2011</td>
<td>20</td>
<td>6.8</td>
<td>5.51E+25</td>
<td>6.4</td>
<td>1.34E+26</td>
<td>Miyagi-ken oki</td>
</tr>
<tr>
<td>S9</td>
<td>15:08 JST 11 Mar 2011</td>
<td>35</td>
<td>7.4</td>
<td>1.40E+27</td>
<td>7.4</td>
<td>3.35E+26</td>
<td>Iwate-ken oki</td>
</tr>
<tr>
<td>S10</td>
<td>15:15 JST 11 Mar 2011</td>
<td>35</td>
<td>7.7</td>
<td>5.66E+27</td>
<td>7.8</td>
<td>6.19E+26</td>
<td>Ibaragi-ken oki</td>
</tr>
<tr>
<td>S11</td>
<td>07:23 JST 28 Mar 2011</td>
<td>20</td>
<td>6.5</td>
<td>1.66E+25</td>
<td>6.1</td>
<td>1.93E+26</td>
<td>Miyagi-ken oki</td>
</tr>
<tr>
<td>S12</td>
<td>20:36 JST 11 Mar 2011</td>
<td>38</td>
<td>6.7</td>
<td>8.33E+25</td>
<td>6.5</td>
<td>3.31E+26</td>
<td>Iwate-ken oki</td>
</tr>
<tr>
<td>A0</td>
<td>23:32 JST 07 Apr 2011</td>
<td>68</td>
<td>7.1</td>
<td>4.74E+26</td>
<td>7.1</td>
<td>1.49E+27</td>
<td>Miyagi-ken oki</td>
</tr>
</tbody>
</table>

*JMA  **F-net by NIED  ***This Study

DATA

We use two sets of data, that is, subduction-zone earthquakes off the Pacific coast and crustal earthquakes around the Fukushima coast. The $A$ of subduction-zone earthquakes are estimated using the $Q$ value for path and empirical amplification factors estimated by a spectral inversion method (Satoh and Tatsumi, 2002) and the strong motion records at K-NET stations. On the other hand, $A$ of crustal earthquakes around the Fukushima coast are estimated by a spectral inversion method in this study because no $Q$ values had been estimated in this region.

Figure 1 shows epicenters by JMA and F-net CMT solutions by NIED of the subduction-zone earthquakes together with location of
K-net strong motion stations by NIED used in this study. The earthquake information is listed in Table 1. The subduction-zone earthquakes are 12 interplate earthquakes with $M_w$ 6.1-7.4 and depth less than 60 km from October 2003 to March 28 2011 and an intraslab earthquake with $M_w$ 7.1 occurred Miyagi-ken oki (off Miyagi prefecture) on April 7, 2011. Records with the peak accelerations of two horizontal components are less than 200 cm/s$^2$ and their S-wave portions with duration of 30 s are used in the analyses. We use data with hypocentral distances less than 180 km for far earthquakes S7 and S8 and the intraslab earthquake A0 and data with hypocentral distances less than 150 km for the other earthquakes.

Figure 2 shows the epicenters by JMA and F-net CMT solutions by NIED of the crustal earthquakes in the Fukushima coast region together with location of K-NET and KiK-net strong motion stations by NIED used in this study. The earthquake information is listed in Table 2. The CMT solution of C8 is not obtained. C7 is strike-slip and the other seven earthquakes are normal earthquakes. Records with the peak accelerations of two horizontal components are less than 200 cm/s$^2$ and their S-wave portions with duration of 20 s for C12 and 10 s for the other 9 crustal earthquakes are used in the analyses.

Strong motion stations located at fore arc region, that is, east of the volcanic front are selected for both datasets because it has been known that short-period waves strongly attenuate through the volcanic front.

**METHOD TO ESTIMATE $A$ OF SUBDUCTION ZONE EARTHQUAKES**

Satoh and Tatsumi (2002) separated a source spectrum $S_i(f)$ of the $i$th event, an attenuation spectrum $P(f)$ and a site-response spectrum $G_j(f)$ at the $j$th station from K-NET records of subduction-zone earthquakes of the Pacific plate in Japan using the generalized spectral
inversion method (Iwata and Irikura, 1988). Therefore an acceleration source spectrum \( O_j(f) \) can be calculated using an observed acceleration spectrum \( F_j(f) \) by

\[
O_j(f) = \frac{F_j(f)}{c \cdot P(f) \cdot G_j(f)} .
\]

(1)

Here

\[
c = \frac{R_0 R_{fS} P_{RTITN}}{4\pi \rho \beta^3} \sqrt{\frac{\rho \beta}{\rho \beta}}
\]

(2)

\[
P(f) = \frac{1}{X} \exp \left( -\frac{\pi X}{Q(f) \beta} \right),
\]

(3)

where \( f \) is the frequency, \( \rho \) and \( \beta \) are the density and S-wave velocity at the source, \( \beta_z \) and \( \rho_z \) are the density and S-wave velocity at a reference site IWT009 (Daito) shown in Fig. 1. \( \beta = 4.0 \) km/s and \( \rho = 3.0 \) g/cm³ are assumed and \( \beta_z \) and \( \rho_z \) are taken to be 2.83 km/s and 2.65 g/cm³ (Satoh and Tatsumi, 2002). \( R_\theta \phi \) is a radiation coefficient and taken to be 0.6 as the mathematical average of radiation patterns for S-waves (Boore and Boatwright, 1984). \( F_s \) is the free surface amplification \( (F_s = 2) \). \( P_{RTITN} \) is the reduction factor that accounts for the partitioning of energy into two horizontal components and taken to be 2 because root-mean-square average of two horizontal spectra is used as a source spectrum. \( X \) is the hypocentral distance and \( Q(f) \) is the unelastic attenuation factor \( Q \) value.

A corner frequency \( f_0 \) of \( i \)th event is estimated to fit \( O_A(f) \) to \( \omega^{-2} \) model \( S(f) \) using revised quasi-Newton method. The misfit \( J(f_0) \) is defined as

\[
J(f_0) = \sum_{j=1}^{N} \int_{f_s}^{f_e} \left( O_j(f) - S_i(f) \right)^2 df ,
\]

(4)

where

\[
S_i(f) = \frac{\left( \frac{2\pi f}{f_0} \right)^2 M_0}{1 + \left( \frac{f}{f_0} \right)^2}.
\]

(5)

The seismic moment \( M_0 \) is taken from CMT solutions (Table 1). \( N \) is the number of station. The minimum frequency \( f_s \) is 0.2 Hz and maximum frequency \( f_e \) is 4 Hz because there is no site-response in this frequency range at IWT009. The \( A \) is calculated from the equation (6) (Brune, 1972; Dan et al., 2001) based on the \( \omega^{-2} \) model.

\[
A = 4\pi \frac{f_0^3}{\lambda} M_0 .
\]

(6)

METHOD TO ESTIMATE \( A \) OF CRUSTAL EARTHQUAKES

The method to estimate \( A \) of crustal earthquakes are the same to Satoh (2010). We first S-wave velocities and damping factors of soils using surface-to-borehole spectral ratios at IBRH14 (Jyuo) assuming one-dimensional wave propagation of vertically incident S-wave by the inversion method by Satoh (2006c) based on an adaptive simulated annealing method (Ingber, 1989; Ingber and Rosen, 1992). Next we separate \( S_i(f), P(f), \) and \( G_j(f) \) from K-NET and KiK-net records using the generalized spectral inversion method (Iwata and Irikura, 1988). Here the theoretical amplification factor for S-wave calculated using inverted soil constants from the seismic bedrock with S-wave velocity of 3.2 km/s to the surface at IBRH14 is used as a restricted condition.

Then \( A \) of each earthquake is estimated by the similar way for subduction-zone earthquakes. \( \beta = 3.5 \) km/s and \( \rho = 2.7 \) g/cm³ are

![Fig. 3. Inversion results of soil constants based on one-dimensional wave propagation theory of S-wave at IBRH14.](image-url)
\( \beta_z \) is taken to be 3.2 km/s based on S-wave logging results at IBRH14. \( \rho_z \) is taken to be 2.65 g/cm\(^3\) based on P-wave logging results and the relation between the P-wave velocity and density (Gardner and Gardner, 1974), respectively.

RESULTS

Figure 3 shows inversion results of soil constants based on one-dimensional wave propagation theory of S-wave at IBRH14. At IBRH14, accelerometers are located on the surface (GL0m) and at a depth of 100 m (GL-100m). The S-wave velocity at GL-100m is 3.2 km/s. Horizontal components of eight earthquakes are used in this inversion analysis. Fig. 3(a) shows observed and inverted surface-to-borehole spectral ratios. Fig. 3(b) shows theoretical amplification factor of S-wave using inverted soil constants shown in Fig. 3(c). In this inversion, damping factor \( h \) of each layer is modeled by

\[
h = h_0 f^{-\alpha} \frac{V_s}{f^\alpha}.
\]

Here \( V_s \) is the S-wave velocity and \( f \) is the frequency. Inverted parameters are common \( a \) of all layers and \( b \) of each layer. The \( a \) is inverted as 0.51. Inverted \( V_s \) and \( h_0 \) are shown in Fig. 3(c).

\( Q \) values for path inverted in this study and previous studies (Satoh and Tatsumi, 2002; Satoh, 2010) are shown in Fig. 4. The \( Q \) for subduction zones of the Pacific plate was estimated as \( Q = 114 f^{-0.92} \) by Satoh and Tatsumi (2002). The \( Q \) for crust around Fukushima coast is estimated as \( Q = 70 f^{-0.9} \) in this study. This value is between \( Q \) of crust in eastern Japan and western Japan estimated by Satoh (2010).

Figure 5 and Figure 6 show the observed acceleration source spectra (equation 1) and \( \omega^{-2} \) models (equation 5) for the intraslab earthquake A0 and 12 interplate earthquakes, respectively. Figure 7 shows the observed acceleration source spectrum and \( \omega^{-2} \) model for the biggest crustal earthquake C10. The agreement between observed and model spectra is reasonably well.

The relations between \( M_0 \) and \( A \) for interplate earthquakes in Iwate-ken oki, Miyagi-ken oki Fukushima-ken oki and Ibaragi-ken oki are shown in Fig. 8. The \( A \) inverted in this study is consistent with the Satoh's (2010) empirical relation for interplate earthquakes of the Pacific plate in Japan. The Satoh's (2010) \( M_0-A \) relation is

\[
A = 4.02 \times 10^{17} M_0^{1/3},
\]

(8)
which is about 1.6 times of Dan et al.'s relation (2001). It is also found that the $A$ of big earthquakes ($M_w > 6.5$) is different among regions. Especially, the $A$ of earthquakes in Miyagi-ken oki tends to be large. The $A$ of the 1976 Miyagi-ken oki earthquake, the 2005 Miyagi-ken oki earthquake and the foreshock S7 have similar $M_o-A$ scaling which is slightly larger than the average + standard deviation of Satoh's (2010) relation.

The $A$ of main shock shown in Fig. 8 is calculated from the characterized source model (Asano and Iwata, 2011; Kamae and Kawave, 2011; Irikura and Kurahashi, 2011) using the following equation (Dan et al., 2001).

$$A = 4\pi \beta^2 \left[ \sum_{i=1}^{N} (\Delta \sigma_i r_i)^2 \right]^{1/2}$$

(9)

Fig. 6. Comparison between observed acceleration source spectra and $\omega^{-2}$ models for 12 interplate earthquakes.

Fig. 7. Comparison between observed acceleration source spectrum and $\omega^{-2}$ model for the biggest normal earthquake C10.
Here $\beta$ is the S-wave velocity of the source, $\Delta\sigma_i$ is the stress drop of $i$th asperity, $r_i$ is the equivalent radius of the $i$th asperity, $N$ is the number of the asperities. Three source models are estimated by the empirical Green's function (EGF) method (Irikura, 1986). We estimate $\beta=3.5\text{km/s}$ for Irikura and Kurahashi (2011) from source parameters of aftershocks and assume 4.0 km/s for Asano and Iwata (2011) and Kamae and Kawabe (2011). The $A$ of Irikura and Kurahashi (2011) is consistent with extrapolation of Satoh's (2010) average relation. The $A$ of Kamae and Kawabe (2011) is consistent with extrapolation of Satoh's (2010) average—standard deviation relation.

In Fig.9 the ratios of $A$ for interplate earthquakes to $A$ by Dan et al. (2001) are shown. It is clearly found that the $A$ of earthquakes in

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**Fig. 8.** $M_0$-$A$ relations for interplate earthquakes on the Pacific plate in this study and previous study estimated by the spectral inversion method

**Fig. 9.** Ratios of $A$ inverted in this study and previous studies for interplate earthquakes to $A$ by the $M_0$-$A$ relation by Dan et al. (2001) for crustal earthquakes.
Miyagi-ken oki is larger than those in the other regions. The $A$ of S12 in Iwate-ken oki is also large, but the mechanism is different from typical thrust earthquakes.

The relations between $M_0$ and $A$ for intraslab earthquakes in Japan are shown in Fig.10, which is revised from Satoh (2004). The $M_0-A$ scaling of the intraslab earthquake $A_0$ shown by the big black circle is similar to that of the Miyagi-ken oki intraslab earthquake in 2003 and is slightly smaller than the $M_0-A$ relation for intraslab earthquakes in the Miyagi-ken oki and the Fukushima-ken oki by Satoh (2004). The Satoh's (2004) $M_0-A$ relation is

$$A = 1.13 \times 10^{18} M_0^{1/3},$$

which is 4.6 times of $A$ by Dan et al.'s (2001) relation. Comparing with the $M_0-A$ scaling of intraslab earthquake in the other regions, $A$

![Image](Fig. 10. $M_0-A$ relations for intraslab earthquakes in Japan in this study and previous study estimated by spectral inversion method.)

![Image](Fig. 11. Comparison of $A$ estimated by different methods. The references on EGF. are shown.)
of the earthquake $A_0$ is smaller than that of the 1994 Hokkaido Toho-oki earthquakes occurring northeast of Iwate-ken oki.

Fig. 11 we compare the $A$ estimated by the spectral inversion method with $A$ calculated from source models estimated by the empirical Green's function (EGF.) method using the equation (9). The $A$ calculated from source models by Morikawa and Sasatani (2004) are revised considering into difference of medium between the source and the reference site ($\rho$/$\rho_z$) based on Sasatani et al. (2006). The $A$ estimated from the different two methods is similar to each other.

The relations between $M_0$ and $A$ for crustal earthquakes estimated by the spectral inversion method are shown in Fig.12. The $A$ of the biggest normal earthquake C10 with $M_w6.6$ is consistent with $A$ of strike-slip earthquakes with $M_w$ of around 6.6, though the $A$ of the second and the third biggest normal earthquakes are slightly smaller than strike-slip earthquakes with $M_w$ of around 5.8. It is also found that both strike-slip and normal earthquakes are smaller than reverse earthquakes with $M_w$ $\approx$ 5.5 in Japan. This result is consistent with the results by Satoh (2006b) and Nakamura (2009) who estimated Brune's stress drops using mainly middle sized crustal earthquakes with $M_w$ 4-6 in Japan. In ground motion attenuation relations by the NGA project (Abrahamson and Silva, 2008; Campbell and Bozorgnia, 2008; Boore and Atkinson, 2008; Chiou and Youngs, 2008), ground motions of reverse earthquakes are the largest, followed in order by strike-slip and normal earthquakes. Our result derived from records of crustal earthquakes in Japan is qualitatively consistent with these attenuation relations derived from records of crustal earthquakes in the world.

CONCLUSIONS

We estimate short-period spectral levels $A$ of acceleration source spectra for aftershocks and foreshocks of the 2011 off the pacific coast of Tohoku earthquake and previous earthquakes in and around this region. The results are summarized as follows:

- The $A$ of the interplate earthquakes estimated in this study are consistent with the Satoh's (2010) $M_0$-$A$ empirical relation for interplate earthquakes of the Pacific plate in Japan. It is also found that the $A$ of big earthquakes occurred in Miyagi-ken oki tend to be large.

- The $A$ of main shock calculated from the source model (Irikura and Kurahashi, 2011) estimated by the empirical Green's function method is consistent with extrapolation of Satoh's (2010) $M_0$-$A$ relation.

- The $A$ of the intraslab earthquake ($M_w$7.1) in the Miyagi-ken oki on 11 April, 2011 is consistent with the Satoh's (2004) empirical relation for intraslab earthquakes in the Miyagi-ken oki and Fukushima-ken oki and is significantly larger than interplate earthquakes in the same regions.

- The $A$ of normal earthquakes around the Fukushima coast which types of big earthquakes had rarely occurred in Japan are slightly smaller than or equal to strike slip earthquakes and significantly smaller than reverse earthquakes in Japan. This tendency qualitatively agrees with ground motion attenuation relations derived from crustal earthquakes in the world by the NGA project. Strong motion predictions would be improved by considering into fault-type and region dependence of $A$ obtained in this study.

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